

Goldstein 4.2

$$(AB)_{ik} = A_{ij} B_{jk}$$

$$\begin{aligned} (AB)_{ik} &= A_{kj} B_{ji} = B_{ji} A_{kj} \\ &= (\tilde{B})_{ij} (\tilde{A})_{jk} \end{aligned}$$

Keep the same formalism for the second part

$$(AB)_{ik} = A_{ij} B_{jk}$$

$$\begin{aligned} (AB)_{ik}^{\dagger} &= (A_{kj} B_{ji})^* \\ &= A_{kj}^* B_{ji}^* \\ &= B_{ji}^* A_{kj}^* \\ &= (B^{\dagger})_{ij} (A^{\dagger})_{jk} \end{aligned}$$